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Non-perturbative SQCD superpotentials from string instantons

Nikolas Akerblom,[a] Ralph Blumenhagen,[a] Dieter Lüst,[ab] Erik Plauschinn[ab] and Maximilian Schmidt-Sommerfeld[a]

[a] *Max-Planck-Institut für Physik*
Föhringer Ring 6, 80805 München, Germany
[b] *Arnold-Sommerfeld-Center for Theoretical Physics, Department für Physik*
Ludwig-Maximilians-Universität München,
Theresienstraße 37, 80333 München, Germany
E-mail: akerblom@mppmu.mpg.de, blumenha@mppmu.mpg.de, luest@mppmu.mpg.de, plausch@mppmu.mpg.de, pumuckl@mppmu.mpg.de

Abstract: The Affleck-Dine-Seiberg instanton generated superpotential for SQCD with $N_f = N_c - 1$ flavours is explicitly derived from a local model of engineered intersecting D6-branes with a single E2-instanton. This computation extends also to symplectic gauge groups with $N_f = N_c$ flavours.

Keywords: D-branes, Intersecting branes models.

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Contents

1. Introduction

Field theory and string theory can both be successfully dealt with in the perturbative regime, where for the latter we essentially only have a perturbative definition of the theory. However, in particular the vacuum structure of both theories depends on the non-perturbative dynamics. In a semi-classical approach such non-perturbative effects are generated by fluctuations around stationary topologically non-trivial field configurations, such as instantons.

In general, these instantons can generate all kinds of effects, but for four-dimensional $\mathcal{N} = 1$ supersymmetric theories their contribution to the holomorphic superpotential can be understood quite explicitly. This has nicely been demonstrated by the celebrated determination of the dynamically generated superpotential for $SU(N_c)$ SQCD with N_f flavours by Affleck, Dine and Seiberg (ADS) [1]. In their paper holomorphy and various local and global symmetries like gauge-, flavour- and R-symmetry were invoked to argue that the superpotential can only have a very peculiar form, whose coefficient was then shown to be non-vanishing for the special case of $SU(2)$.[1]

As mentioned above, instanton effects are also very important for an understanding of the vacuum structure (these days called the landscape) of string compactifications. Even though we do not have a non-perturbative definition of the theory, by analogy to field theory rules for dealing with such string instantons have been elaborated on [3 – 14]. The field theory techniques are described in [15, 16] (and references therein). These instantons are given by wrapped string world-sheets and by wrapped Euclidean D-branes and, like in field theory, their contributions to the space-time superpotential are quite restricted. These contributions can be computed in a semi-classical approach, i.e. one involving only the tree level instanton action and a one-loop determinant for the fluctuations around the instanton. Recently, in the KKLT scenario [17], wrapped D3-brane instantons have also played a major role in eventually fixing all moduli. For concrete orientifold realisations of KKLT see [18 – 20].

For type IIA orientifold models on Calabi-Yau spaces with intersecting D6-branes (and their T-dual cousins) the contribution of wrapped Euclidean D2-branes, hereafter called E2-branes, to the superpotential has been determined in [21] (see also [22 – 24]). Since both the D6-branes and the E2-instantons are described by an open string theory, it was shown that the entire instanton computation boils down to the evaluation of disc and 1-loop string diagrams with boundary (changing) operators inserted. Here both the D6-branes and the E2-instantons wrap compact three-cycles of the Calabi-Yau manifold. Note that in [21] the instanton calculus was developed for the case that the instantons do not lie on top of any stack of D6-branes, implying that only charged fermionic zero modes appear.

The natural question now arises whether one can consistently recover known field theory instanton effects like the ADS superpotential from string theory. Recall that $\mathcal{N}=1$ SQCD with gauge group $SU(N_c)$ and N_f flavours, Φ_f, $\widetilde{\Phi}_{f'}$ with $N_f = N_c - 1$ does indeed have a non-perturbatively generated superpotential. In [1] the authors argued that in this case the superpotential is generated by a single field theory instanton and using various symmetries they restricted the form of the superpotential to be

$$W = \frac{\Lambda^{3N_c - N_f}}{\det[M_{ff'}]} \,,$$

with the meson matrix of rank N_f defined as $M_{ff'} = \Phi_f^c \, \widetilde{\Phi}_{c,f'}$. The above question has been posed in [25] for the case of the runaway quiver gauge theory of [26]. There the gauge theory was $SU(3)$ with two quarks and the stringy origin of the constraint $N_f = N_c - 1$ was not really obvious. The above question also relates to recent attempts at freezing string moduli via flux induced superpotentials, often extended by known non-perturbative

[1] cf. also the explicit determination of prefactors by Cordes [2].

superpotentials, which were actually only derived for supersymmetric Yang-Mills theories. For such a simple addition of these terms to make any sense, one must ensure that string theory indeed does generate these field theory superpotentials by itself. Of course, one expects that the field theory result can only be obtained in certain limits of string theory, so one must know under what circumstances one can trust them.

Since such effects scale like $\exp(-1/g^2_{SU(N_c)})$ in field theory, it is clear that the E2-brane has to lie on top of the $U(N_c)$ stack of D6-branes. In this case additional bosonic zero modes appear which have to be integrated over. An E2-instanton inside a D6-brane is very much like the D3-D(-1) system, which is known to give a stringy realization of the ADHM instanton moduli space in the field theory limit [27].

In this paper we show that applying the general formalism of [21] to a locally engineered system of colour and flavour D6-branes with an E2-instanton inside the colour branes indeed gives the ADS superpotential in the field theory limit. Let us emphasise that this result provides an important check on the rules proposed in [21]. We note that we are not using any a priori assumptions to fix the shape of the superpotential, but carry out the non-trivial evaluation of the zero mode integrals in full glory. To our knowledge, such a direct computation has so far only been carried out in [2], though with a different instanton zero mode measure than the ADHM measure which arises naturally in string theory. As already used in [25], the D-brane realisation of both the gauge theory and the instanton directly leads to extended quiver diagrams containing both the D6-branes and the E2-instantons. This allows one to diagrammatically encode both the massless spectrum including the instanton zero modes and possible allowed disc correlators directly in the quiver diagrams. We easily recover the $N_f = N_c - 1$ constraint from such a counting of fermionic zero modes.

We also apply our techniques to the case of $USp(2N_c)$ SQCD with N_f flavours and again obtain the known field theory result for the dynamically generated superpotential [28]. We comment also on the slightly more involved $SO(N_c)$ case [29], which, in principle, is also amenable to our methods.

This paper is organised as follows: In section 2 we first briefly recall the main results of [21], which provide a recipe of how to compute contributions to the superpotential by E2-brane instantons in a conformal field theory framework. In addition we derive some new results on the generalisation of this framework to E2-instantons carrying extra bosonic zero modes. Moreover, we make a concrete proposal relating the one-loop determinants for the fluctuations around the instanton to one-loop gauge threshold corrections. Then we specify which E2-instantons can be related to field theory gauge instantons in a certain field theory limit of string theory. That means, we locally engineer a brane set-up which corresponds to SQCD. In section 3 we compute the relevant disc amplitudes of the various boundary changing operators involving both the matter and the instanton zero modes. In section 4 we explicitly carry out the instanton zero mode integration, which turns out to be non-trivial, and eventually in the field theory limit we indeed arrive at the ADS superpotential. In section 5 this result is generalised to supersymmetric $USp(2N_c)$ gauge theory with N_f flavours and in section 6 we give our conclusions.

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2. E2-instantons in type II D-brane models

Let us review and slightly extend the formalism developed in [21] for the computation of instanton contributions to the superpotential of Type IIA orientifolds with intersecting D6-branes and E2-instantons wrapping compact three-cycles of the internal Calabi-Yau geometry. We could also work with the possible T-dual Type IIB configurations, but in order to have a clear geometric picture of the arising open string zero modes we choose the Type IIA framework. We first discuss a compact set-up and then, as usual for string engineering of field theories, take a decoupling limit to eventually arrive at local geometries comprising all the information relevant for the field theory. For the techniques of model building with intersecting brane worlds in general, see for instance [30 − 32].

2.1 Instanton generated type IIA superpotential

Assume we have a Type IIA orientifold with O6-planes and intersecting D6-branes preserving $\mathcal{N} = 1$ supersymmetry in four dimensions, i.e. the D6-branes wrap special Lagrangian (sLag) three-cycles Π_a of the underlying Calabi-Yau manifold, all preserving the same supersymmetry.

Space-time instantons are given also by D-branes, which in this case are Euclidean E2-branes wrapping three-cycles Ξ in the Calabi-Yau, so that they are point-like in four-dimensional Minkowski space. Such instantons can contribute to the four-dimensional superpotential, if they preserve half of the $\mathcal{N} = 1$ supersymmetry, which means they also wrap sLag three-cycles. This guarantees that there are at least the four translational zero modes x_μ and two fermionic zero modes θ_i arising from the two broken supersymmetries. If the instanton is not invariant under the orientifold projection there are another two fermionic zero modes $\overline{\theta}_i$, as the instanton breaks four of the eight supercharges in the bulk [33]. We also require that there do not arise any further zero modes from E2-E2 and E2-E2' open strings, so that in particular the three-cycle Ξ should be rigid, i.e. $b_1(\Xi) = 0$.

Therefore, considering an E2-instanton in an intersecting brane configuration, additional zero modes can only arise from the intersection of the instanton Ξ with D6-branes Π_a:

- At any rate, there are $N_a\, [\Xi \cap \Pi_a]^+$ chiral fermionic zero modes $\lambda_{a,I}$ and $N_a\, [\Xi \cap \Pi_a]^-$ anti-chiral ones, $\widetilde{\lambda}_{a,J}$.[2]
- At a smooth point in the CY moduli space, in general there will be no bosonic zero modes originating form the instanton. However, if the instanton lies right on top of a D6-brane one gets $4\, N_a$ bosonic zero modes. Let us organize them into two complex modes b_α with $\alpha = 1, 2$.

As we will see, for $\Xi = \Pi_a$ there can appear extra conditions on the zero modes arising from the flatness conditions for the effective zero-dimensional gauge theory on the E2-instanton. In this case the above numbers only give the unrestricted number of zero modes.

[2]Here we introduced the physical intersection number between two branes $\Pi_a \cap \Pi_b$, which is the sum of positive $[\Pi_a \cap \Pi_b]^+$ and negative $[\Pi_a \cap \Pi_b]^-$ intersections.

In [27, 21] it was argued that each E2-D6 zero mode carries an extra normalisation factor of $\sqrt{g_s}$ so that contributions to the superpotential can only arise from CFT disc and one-loop diagrams involving E2 as a boundary. Here the disc contains precisely two zero mode insertions and the 1-loop diagram none. For details we refer the reader to [21], here we only recall the final result.

For its presentation it is useful to introduce the short-hand notation

$$\widehat{\Phi}_{a_k,b_k}[\vec{x}_k] = \Phi_{a_k,x_{k,1}} \cdot \Phi_{x_{k,1},x_{k,2}} \cdot \Phi_{x_{k,2},x_{k,3}} \cdot \ldots \cdot \Phi_{x_{k,n-1},x_{k,n}} \cdot \Phi_{x_{k,n(k)},b_k} \tag{2.1}$$

for the chain-product of open string vertex operators. We define $\widehat{\Phi}_{a_k,b_k}[\vec{0}] = \Phi_{a_k,b_k}$. The single E2-instanton contribution to the superpotential can be determined by evaluating the following zero mode integral over disc and 1-loop open string CFT amplitudes

$$\begin{aligned} S_W =& \frac{1}{\ell_s^3} \int d^4x\, d^2\theta \sum_{\text{conf.}} \prod_I d\lambda_I \prod_J d\widetilde{\lambda}_J \prod_{\alpha=1}^2 \prod_i db_{\alpha,i}\, d\overline{b}_{\alpha,i} \\ & \times \delta(\lambda, \widetilde{\lambda}, b_\alpha, \overline{b}_\alpha) \times \exp\big(\langle 1\rangle^{\text{1-loop}}\big) \times \exp\Big(\langle 1\rangle^{\text{disc}}\Big) \\ & \times \exp\left(\sum_k \langle\langle \Phi_{a_k,b_k}[\vec{x}_k]\rangle\rangle^{\text{disc}}_{\lambda_k\widetilde{\lambda}_k}\right) \times \exp\left(\sum_l \langle\langle \Phi_{a_l,b_l}[\vec{x}_k]\rangle\rangle^{\text{disc}}_{b_{\alpha,l}\overline{b}_{\alpha,l}}\right) , \end{aligned} \tag{2.2}$$

where we work in the convention that all fields carry no scaling dimension and the sum is over all possible vertex operator insertions. Note that in this formula the $\overline{\theta}_i$ zero modes have been integrated over leading to δ-functions representing the fermionic ADHM constraints [34], as will be explained later on. There are further δ-functions involving only bosonic zero modes. These incorporate the bosonic ADHM constraints or, in more physical terms, the F-term and D-term constraints from the effective theory on the E2-brane. Since we will not attach any charged matter fields to the one-loop diagrams, compared to [21], we left out this term in (2.2). The prefactor ℓ_s^{-3} is chosen for dimensional reasons and is so far only determined up to numerical factors. (We will come back to such normalisation issues later.) The vacuum disc amplitude is given by

$$\langle 1\rangle^{\text{disc}} = -S_{\text{inst}} = -\frac{1}{g_s}\frac{V_{\text{inst}}}{\ell_s^3} = -\frac{8\pi^2}{g^2_{\text{YM}}} , \tag{2.3}$$

where the four-dimensional Yang-Mills gauge coupling is meant to be for the gauge theory on a stack of D6-branes wrapping the same internal three-cycle as the instanton. As already mentioned, in [21] the superpotential was given for the case without any additional charged bosonic zero modes b_α, $\overline{b}_\alpha$. Since they also carry the extra normalisation factor $\sqrt{g_s}$, precisely two of them can be attached to a disc diagram. Moreover, since they commute the whole exponential term appears in the superpotential (2.2).

The one-loop contributions are annulus diagrams for open strings with one boundary on the E2-instanton and the other boundary on the various D6-branes

$$\langle 1\rangle^{\text{1-loop}} = \sum_a \Big[Z'^A(\text{E2}, \text{D6}_a) + Z'^A(\text{E2}, \text{D6}'_a)\Big] + Z'^M(\text{E2}, \text{O6}) . \tag{2.4}$$

Here Z' means that we only sum over the massive open string states in the loop amplitude, as the zero modes are taken care of explicitly. Without soaking up these zero modes, one clearly would encounter divergences from integrating out bosonic zero modes and zeros from integrating out fermionic zero modes. We have provided the formula for an orientifold model on a compact Calabi-Yau manifold. For the local engineering of SQCD done later in this paper, we do not really have to take the orientifold planes and the image branes into account, so that the annulus amplitudes with the image branes and the Möbius strip amplitudes are absent.

The last ingredients to compute the rhs of equation (2.2) are the disc diagrams with insertion of matter fields $\Phi_{a,b}$ and fermionic or bosonic zero modes. In the case of inserting fermionic zero modes λ and $\widetilde{\lambda}$, the explicit form of the disc diagram is the following

$$\langle\langle \Phi_{a_k,b_k}[\vec{x}_k]\rangle\rangle^{\text{disc}}_{\lambda_k\widetilde{\lambda}_k} = \int \frac{dz_1 \ldots dz_{n+2}}{V_{1,2,n+2}} \, \langle V_{\lambda_k}(z_1)\, V_{\Phi_{a_k,x_{k_1}}}(z_2) \, \ldots \, V_{\Phi_{x_{k_n},b_k}}(z_{n+1})\, V_{\widetilde{\lambda}_k}(z_{n+2})\rangle \; . \tag{2.5}$$

For the disc diagram with insertion of bosonic zero modes b_α and $\overline{b}_\alpha$, one simply replaces $V_{\lambda_k}(z_1)$ by $V_{b_{\alpha,k}}(z_1)$ and $V_{\widetilde{\lambda}_k}(z_{n+2})$ by $V_{\overline{b}_{\alpha,k}}(z_{n+2})$. In equation (2.5) the symbol V_{ijk} denotes the measure

$$V_{ijk} = \frac{dz_i\, dz_j\, dz_k}{(z_i - z_j)(z_i - z_k)(z_j - z_k)} \tag{2.6}$$

with $z_i \to \infty$, $z_j = 1$ and $z_k = 0$. The remaining z's are to be ordered as $1 \geq z_3 \geq z_4 \geq \ldots \geq z_{n+1} \geq 0$ and integrated over the interval $[0, 1]$.

To summarise, the entire computation of the single instanton contribution to the superpotential boils down to the following steps:

- Determining the bosonic and fermionic instanton zero modes from all possible open string sectors.
- Finding the E2-flatness contraints for instantons sitting on top of D6-branes and implementing them as delta-functions in (2.2).
- The evaluation of certain disc amplitudes with insertion of precisely two instanton zero modes and an arbitrary number of matter fields (see equation (2.5)).
- The computation of vacuum annulus and Möbius strip amplitudes as indicated in equation (2.4).
- Performing the integration over bosonic and fermionic zero modes.

As expected, the stringy superpotential is entirely given in a semi-classical approximation. We will see that in the field theory limit, only the leading order terms of the disc and one-loop amplitudes survive, considerably simplifying formula (2.2).

2.2 One-loop amplitudes

So far we have not really specified what we actually mean by the vacuum one-loop amplitudes, i.e. the annulus amplitudes for open strings between the E2-instanton and the

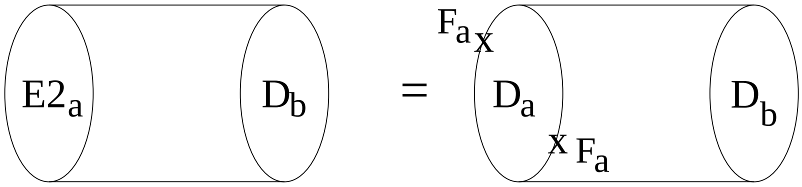

Figure 1: Relation between instantonic one-loop amplitudes and corresponding gauge threshold corrections.

various D6-branes. To our knowledge such amplitudes have not been computed yet. So far, only overlaps of two non-instantonic or two instantonic [35, 8, 36] D-branes have been considered in the literature.[3]

In order to get an idea beforehand what they should be, we first note that the divergence from the massless open string excitations should be identified with the divergence from integrating out naively the instanton zero modes, i.e. without soaking them up in disc diagrams involving other space-time fields. Next we observe that for the case in which we place an $\mathrm{E2}_a$-instanton on top of a $\mathrm{D6}_b$-brane, the disc instanton action

$$S_{\rm inst} = \frac{8\pi^2}{g^2_{\rm YM}} \tag{2.7}$$

is equal to the tree level gauge coupling on $\mathrm{D6}_b$. This gauge coupling receives one-loop threshold corrections from open strings between the brane $\mathrm{D6}_a$ and the branes $\mathrm{D6}_b$ of the form [38]

$$\frac{8\pi^2}{g_a^2(\mu)} = \frac{8\pi^2}{g_a^2(\mu_0)} + \sum_b b^a_{[ab]}\, \ln\left(\frac{\mu}{\mu_0}\right) + \Delta^a_{[ab]}, \tag{2.8}$$

where $b^a_{[ab]}$ is the field theory one-loop beta function coefficient and comes from massless states of the $\mathrm{D6}_a$-$\mathrm{D6}_b$ open string sector running in the loop. To make the picture consistent, it should better be that the one-loop contributions $Z^A(\mathrm{E2}_a, \mathrm{D6}_b)$ are identical to the one-loop gauge threshold corrections $T^a(\mathrm{D6}_a, \mathrm{D6}_b)$. More concretely, the sum of the three even spin structures will be shown to yield the one-loop correction to the gauge coupling, and the odd spin structure is expected to give the one-loop correction to the θ-angle. We therefore propose that diagrammatically we have the intriguing relation shown in figure 1.

To prove this statement let us first compute the instantonic one-loop partition functions $Z^A(\mathrm{E2}_a, \mathrm{D6}_b)$. These cannot be expressed in light cone gauge, but including the contributions from the ghosts can, for the even spin structures, be written as

$$Z^A(\mathrm{E2}_a, \mathrm{D6}_b) = -\int_0^\infty \frac{dt}{t} \sum_{\alpha,\beta\neq(\frac{1}{2},\frac{1}{2})} (-1)^{2(\alpha+\beta)}\, \frac{\Theta^2\left[{\alpha \atop \beta}\right](it, it/2)}{\Theta^2\left[{1/2 \atop 1/2}\right](it, it/2)}\, \frac{\eta^3(it)}{\Theta\left[{\alpha \atop \beta}\right](it, 0)}\, A^{\rm CY}_{ab}\left[{\alpha \atop \beta}\right] \tag{2.9}$$

[3] Just when the present paper was readied for submission, we received [37], where these E2-D6 partition functions were computed for intersecting branes on $\mathbb{T}^6$.

with

$$\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](\tau,z) = \sum_{n\in\mathbb{Z}} e^{i\pi\tau(n+\alpha)^2}\, e^{2\pi i(n+\alpha)(z+\beta)}\,. \tag{2.10}$$

Moreover, $A_{ab}^{\rm CY}\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right]$ denotes the internal open string partition function with the respective spin-structure for open strings between branes wrapping the three-cycles Π_a and Π_b. Using the relations

$$\left(\frac{\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](it,z)\,\Theta'\left[\begin{smallmatrix}1/2\\ 1/2\end{smallmatrix}\right](it,0)}{\Theta\left[\begin{smallmatrix}1/2\\ 1/2\end{smallmatrix}\right](it,z)\,\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](it,0)}\right)^2 = \frac{\Theta''\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](it,0)}{\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](it,0)} - \partial_z^2 \log\Theta\left[\begin{smallmatrix}1/2\\ 1/2\end{smallmatrix}\right](it,z) \tag{2.11}$$

where the derivatives are taken with respect to the variable z, and $-2\pi\eta^3(it) = \Theta'\left[\begin{smallmatrix}1/2\\ 1/2\end{smallmatrix}\right](it,0)$, after a few standard manipulations we can bring the partition function into the form

$$Z^A(\mathrm{E2}_a,\mathrm{D6}_b) = \int_0^\infty \frac{dt}{t} \sum_{\alpha,\beta\neq(\frac12,\frac12)} (-1)^{2(\alpha+\beta)}\,\frac{\Gamma\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](it)}{\eta^3(it)}\, A_{ab}^{\rm CY}\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right]\,. \tag{2.12}$$

The Γ's are nothing else than the derivatives of Θ-functions with respect to the variable t

$$\Gamma\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](it) = -\frac{1}{\pi}\frac{\partial}{\partial t}\Theta\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right](it,0) = \sum_{n\in\mathbb{Z}} (n+\alpha)^2\, e^{-\pi t(n+\alpha)^2}\, e^{2\pi i(n+\alpha)\beta}\,. \tag{2.13}$$

The threshold corrections for intersecting D6-branes on a torus $\mathbb{T}^6$ have been explicitly computed in [38]. Their result is easily generalised to the case of intersecting D6-branes on a general Calabi-Yau space and precisely gives (2.12).

Now, divergences in (2.12) can arise from massless states. As we mentioned, these divergences are precisely related to the one-loop beta-function coefficient $b^a_{[ab]}$ for the $\mathcal{N}=1$ supersymmetric massless states between the branes $\mathrm{D6}_a$ and $\mathrm{D6}_b$. This one-loop divergence is roughly

$$\exp\left(b^a_{[ab]}\int_{t_0}^t \frac{dt}{t} q^0\right) = \left(\frac{t}{t_0}\right)^{b^a_{[ab]}} = \left(\frac{\mu_0}{\mu}\right)^{b^a_{[ab]}}\,, \tag{2.14}$$

where we have defined the mass scales $\mu = 1/t$ and $\mu_0 = 1/t_0$. The beta function coefficient $b^a_{[ab]}$ has contributions from the NS- and the R-sector in (2.12)

$$b^a_{[ab]} = \left(b^a_{[ab]}\right)_{NS} - \left(b^a_{[ab]}\right)_R\,. \tag{2.15}$$

Identifying, as proposed, $T^a(\mathrm{D6}_a,\mathrm{D6}_b)$ with $Z^A(\mathrm{E2}_a,\mathrm{D6}_b)$, these divergences should come from integrating out bosonic and fermionic instanton zero-modes. Taking into account that a mass term in the action for a boson reads $\mathcal{L}^B_{\rm mass} = \mu^2 b\,\overline{b}$ and for a fermion $\mathcal{L}^F_{\rm mass} = \mu\,\lambda\,\widetilde{\lambda}$, the number of bosonic and fermionic zero modes should be identified with

$$n_B = \left(b^a_{[ab]}\right)_{\rm NS}\,, \quad n_F = 2\,\left(b^a_{[ab]}\right)_{\rm R}\,. \tag{2.16}$$

Let us verify that this indeed gives the correct number of instanton zero modes for the configurations relevant in the following:

- $\underline{a \neq b}$: Let us discuss the case that we have an intersection giving rise to one pair of chiral superfields between the D6$_a$ and a stack of N_b D6$_b$-branes. In the NS sector, for the massless states the internal sector carries $h = 1/2$ and the external one only the ground state with $h = 0$. However, in the threshold corrections this latter $n = 0$ state does not survive and therefore all states from the NS-sector are massive and $(b^a_{[ab]})_{\rm NS} = 0$. In the R-sector with $\alpha = 1/2$, the $(n + \alpha)^2$ factor implies that we get $(b^a_{[ab]})_{\rm R} = N_b$, leading to the correct beta-function coefficient for N_b generations of quarks. The number of fermionic zero modes for the E2$_a$ instanton and the N_b D6-branes is therefore

$$n_B = 0\,, \qquad n_F = 2\,N_b\,. \tag{2.17}$$

- $\underline{a = b}$: Now let us discuss the case that we put the instanton E2$_a$ on top of the stack of D6$_a$-branes. If the D6$_a$ brane is rigid, we get precisely one $\mathcal{N} = 1$ vector superfield in the corresponding D6$_a$-D6$_a$ open string sector. These are the states with $n = \pm 1$ in the external sector. Therefore, in this case we find $(b^a_{[aa]})_{\rm NS} = 4\,N_a$ in (2.12). The fermionic contribution comes out as $(b^a_{[aa]})_{\rm R} = N_a$, giving the correct beta-function coefficient for one $\mathcal{N} = 1$ vector field $b^a_{[aa]} = 3\,N_a$. The number of instanton zero modes therefore is as expected

$$n_B = 4\,N_a\,, \qquad n_F = 2\,N_a\,. \tag{2.18}$$

We see that in these two examples indeed the E2-D6 partition functions are identical to the gauge threshold correction in the corresponding D6-D6 sector.[4]

2.3 Engineering of field theory instantons

The formalism presented so far deals with truly string instantons for compact Type IIA orientifolds. In the following we would like to extract the instanton amplitudes for field theory instantons in supersymmetric Yang-Mills theories. This means that we have to decouple gravity and higher order string corrections. Field theory instantons in a supersymmetric Yang-Mills theory contain a factor $\exp(-1/g^2_{\rm YM})$, which immediately implies that such effects can only arise from E2-branes sitting right on top of the D6-brane supporting the Yang-Mills theory.

To be concrete, let us engineer, in the spirit of [39, 40], $SU(N_c)$ supersymmetric Yang-Mills theory with N_f non-chiral flavours. This theory is generally called SQCD. First, we take a stack of N_c D6-branes wrapping a three-cycle Π_c of some Calabi-Yau manifold. In order to just get $\mathcal{N} = 1$ super Yang-Mills theory on this brane, we assume that the three-cycle is special Lagrangian and rigid, i.e. that the number of its deformations satisfies $b_1(\Pi_c) = 0$. In D-brane models, fundamental representations of matter fields can only

[4] Note that the superpotential in the Wilsonian sense is a holomorphic quantity but the stringy gauge threshold corrections in eq. (2.12) generically are not. Therefore, only the Wilsonian holomorphic part of the threshold corrections enters the instanton generated superpotential.

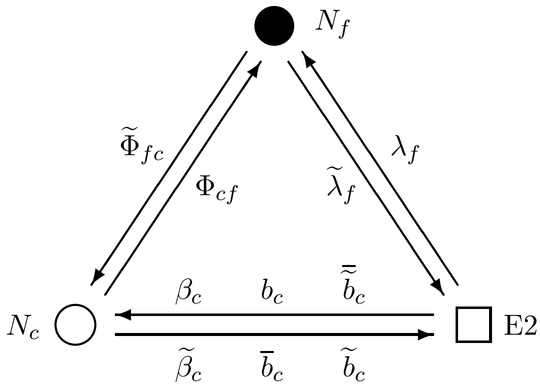

Figure 2: Local SQCD brane configuration with resulting zero modes.

come from bi-fundamental representations arising from the intersection with a stack of N_f flavour branes. These flavour branes wrap a different sLag three-cycle Π_f. As we will discuss in subsection 2.4, we would like the $U(N_f)$ gauge dynamics on these branes to decouple.

Since we want non-chiral matter,[5] we assume that the intersection number of Π_c and Π_f is non-chiral, which means

$$[\Pi_c \cap \Pi_f]^{\pm} = 1 \; . \tag{2.19}$$

With this particle content, all cubic gauge anomalies vanish so that we can assume that these two D-branes do not have any other non-trivial intersection with the other potentially present D6-branes in the model. Generically the gauge boson in the $U(1) \subset U(N_c)$ subgroup acquires a mass due to GS mixing terms with the axions.

As we mentioned above, in string theory, the (zero size) Yang-Mills instanton in $SU(N_c)$ is given by a Euclidean E2-brane wrapping the same three-cycle Π_c as the colour branes and being point-like in four-dimensional Minkowski space. The final local D6-E2 brane configuration and the resulting zero modes are shown in the extended quiver diagram in figure 2.

In this local model, there are three open string sectors involving the instanton. We analyse them in turn.

E2-E2: since the E2-instanton wraps the same rigid three-cycle as the D6$_c$ brane, the massless E2-E2 modes are the four positions in Minkowski space x_μ with $\mu = 0, \ldots, 3$ and four massless fermions θ_i, $\overline{\theta}_i$, $i = 1, 2$. The latter correspond to the breaking of four of the eight supercharges preserved in type IIA theory on a Calabi-Yau manifold. Two of these, namely the θ_i, are related to the two supersymmetries preserved by the D6-branes, but

[5]In toroidal $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$ compactifications with intersecting branes this can be achieved for instance by choosing intersection angles $\phi_1 = 0$ and $\phi_2 = -\phi_3$ between Π_c and Π_f.

broken by the instanton, and, together with the four bosons x_μ contribute the measure

$$\int d^4x\, d^2\theta \tag{2.20}$$

to the instanton amplitude. Integrating the other two zero modes $\overline{\theta}_i$ leads to the fermionic ADHM [34] constraints [27, 41] as we will explain in the following.

E2-D6$_c$: since the E2-instanton and the D6$_c$ branes wrap the same three-cycle, they share three directions in the internal space and have N-D boundary conditions along Minkowski space. Therefore, the ground state energy in both the NS- and the R-sector is $E_{\mathrm{NS,R}} = 0$ and one gets N_c complete hypermultiplets of massless modes. These are the $4\,N_c$ bosonic zero modes already seen in eq. (2.18).

However, not all of these bosonic zero modes are independent. In fact as analysed in detail in [27], the effective zero-dimensional supersymmetric gauge theory on the E2-brane gives rise to D-term and F-term constraints, which in the field theory limit are nothing else than the well known ADHM [34] constraints [42]. If we rename the complex bosonic zero modes $b_{\alpha,i}$ as $b_{1,i} = b_i$ and $b_{2,i} = \overline{\widetilde{b}}_i$, these constraints become [43]

$$\sum_{c=1}^{N_c} \left[b_c\, \widetilde{b}_c + \overline{b}_c\, \overline{\widetilde{b}}_c \right] = 0\,, \quad \sum_{c=1}^{N_c} \left[b_c\, \widetilde{b}_c - \overline{b}_c\, \overline{\widetilde{b}}_c \right] = 0\,, \quad \sum_{c=1}^{N_c} \left[b_c\, \overline{b}_c - \widetilde{b}_c\, \overline{\widetilde{b}}_c \right] = 0\,. \tag{2.21}$$

The first two can be interpreted as the F-term constraints Re $b_c\widetilde{b}_c = 0$ and Im $b_c\widetilde{b}_c = 0$, the last one as the D-term constraint $\sum_{c=1}^{N_c} |b_c|^2 - |\widetilde{b}_c|^2 = 0$.

As seen in (2.18), there are only $2N_c$ fermionic zero modes. However, in [27] it was shown in the T-dual picture that they have to satisfy the ADHM constraints

$$\sum_{c=1}^{N_c} \left[\widetilde{\beta}_c\, b_c + \beta_c\, \widetilde{b}_c \right] = 0 \qquad \text{and} \qquad \sum_{c=1}^{N_c} \left[\widetilde{\beta}_c\, \overline{\overline{b}}_c - \beta_c\, \overline{b}_c \right] = 0\,. \tag{2.22}$$

These constraints can be recovered in the present setup by performing the integration over the aforementioned zero modes $\overline{\theta}_i$ explicitly [27, 41]. More concretely, one can absorb the instanton zero modes with terms like $\tilde{\beta}_c\, b_c\, \overline{\theta}$ and integrate over $\overline{\theta}$ to arrive at a δ-function realisation of (2.22).

All these constraints, i.e. equations (2.21) and (2.22), have to be implemented in the general formula for the superpotential (2.2). We will not perform the $\overline{\theta}$ integration explicitely but realise all the constraints by the means of delta functions. This ensures the gauge invariance of the integration measure.

E2-D6$_f$: in this sector, the ground state energy in the NS sector is positive, so that there are no bosonic zero modes. One only gets N_f pairs of non-chiral $\lambda_f, \widetilde{\lambda}_f$ zero modes from eq. (2.17) yielding the measure

$$\int \prod_{f=1}^{N_f} d\lambda_f\, d\widetilde{\lambda}_f\,. \tag{2.23}$$

To summarise, the total integration measure for the computation of the E2-instanton generated superpotential (2.2) reads

$$\int d^4x\, d^2\theta \prod_{c=1}^{N_c} db_c\, d\overline{b}_c\, d\widetilde{b}_c\, d\overline{\widetilde{b}}_c d\beta_c\, d\overline{\beta}_c \prod_{f=1}^{N_f} d\lambda_f\, d\widetilde{\lambda}_f\;\; \delta^F(\widetilde{\beta}_c\, b_c + \beta_c\, \widetilde{b}_c) \\ \delta^F(\widetilde{\beta}_c\, \overline{\widetilde{b}}_c - \beta_c\, \overline{b}_c)\; \delta^B(b_c\, \widetilde{b}_c + \overline{b}_c\, \overline{\widetilde{b}}_c)\; \delta^B(b_c\, \widetilde{b}_c - \overline{b}_c\, \overline{\widetilde{b}}_c)\; \delta^B(b_c\, \overline{b}_c - \widetilde{b}_c\, \overline{\widetilde{b}}_c)\;, \tag{2.24}$$

where in the two fermionic and three bosonic ADHM constraints summation over the colour index is understood. It is rather amusing that the open string zero modes precisely give the flat ADHM measure with the quadratic constraints arising from the F-term and D-term constraints for the effective gauge theory on the E2-instanton.

2.4 The field theory limit

Now let us discuss the aforementioned field theory limit in more detail. We have a stack of colour D6-branes wrapping a three-cycle of size

$$v_c = \frac{V_c}{\ell_s^3} \cong \frac{R_c^3}{(\alpha')^{\frac{3}{2}}} = r_c^3 \tag{2.25}$$

in string units. The flavour branes wrap a different cycle with volume $v_f = r_c\, r_T^2$, where T stands for transversal.[6] The volume of the entire CY in string units is $v_{\rm CY} \cong r_c^3\, r_T^3$ and we have two dimensionless scales r_c, r_T and one length scale $\sqrt{\alpha'}$.

The effective four-dimensional couplings in our configuration are the Yang-Mills coupling of the colour branes

$$\frac{4\pi}{g_c^2} = \frac{1}{g_s}\frac{V_c}{\ell_s^3} \cong \frac{r_c^3}{g_s}\;, \tag{2.26}$$

the Yang-Mills coupling of the flavour branes

$$\frac{4\pi}{g_f^2} = \frac{1}{g_s}\frac{V_f}{\ell_s^3} \cong \frac{r_c\, r_T^2}{g_s} \tag{2.27}$$

and the Planck mass

$$M_{\rm Pl}^2 \cong \frac{1}{(2\pi\alpha')\, g_s^2} r_c^3\, r_T^3\;. \tag{2.28}$$

Clearly, the gauge coupling of the colour branes should be fixed at finite g_c. In order for the Dirac-Born-Infeld theory on these branes to reduce to the Yang-Mills Lagrangian we have to suppress all higher α' corrections, which means we have to take the $\alpha' \to 0$ limit.

On the other hand the gauge theory on the flavour branes should decouple from the dynamics, which means that $g_f \to 0$ or, in other words, that the transverse space of volume $v_T = r_T^3$ should decompactify, i.e. $r_T \to \infty$. In this limit however, also $M_{\rm Pl} \to \infty$ so that gravity decouples.

[6] We are thinking here of the situation on the six-torus, where the flavour- and the colour-cycle share one common direction.

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In this limit the world-sheet instantons, giving stringy corrections to the disc correlators, scale like

$$\exp\left(-\frac{J}{\alpha'}\right) \simeq \exp(-r_T^2) \to 0 \,, \tag{2.29}$$

so that only trivial instantons of zero size contribute to the field theory amplitude. Moreover, in the one-loop Pfaffian all the massive string modes decouple in the $\alpha' \to 0$ limit. The same holds for potential Kaluza-Klein and winding modes in the various E2-D6 brane sectors, as their masses scale like

$$M_{\rm KK}^2 = \frac{1}{R_c^2} \simeq \frac{1}{\alpha' r_c^2} \to \infty \qquad \text{and} \qquad M_{\rm wind}^2 = \frac{R_f^2}{\alpha'^2} \simeq \frac{r_T^2}{\alpha'} \to \infty \,. \tag{2.30}$$

The gauge coupling of the effective zero-dimensional gauge theory on the E2-brane reads

$$\frac{4\pi}{g_E^2} = \frac{\ell_s^4}{g_s}\frac{V_3}{\ell_s^3} \simeq \frac{\alpha'^2\, r_c^3}{g_s} \to 0 \,, \tag{2.31}$$

i.e. naively, in the field theory limit the effective theory on the instanton decouples. However, this theory should provide the ADHM constraints, so that it should better not decouple completely. In fact, as shown in [27], the operators up to dimension four in the E2-action survive precisely due to extra factor of $\sqrt{g_E} \simeq \sqrt{g_s}$ in the vertex operators for the E2-D6 modes.

To summarise, in the field theory limit,

$$\alpha' \to 0 \,, \qquad v_T \to \infty \,, \qquad v_c = \text{finite} \,, \tag{2.32}$$

the complete string theory instanton amplitude simplifies dramatically and only the leading order world-sheet disc instantons and massless states in the 1-loop amplitudes contribute. Moreover, all higher order α' corrections in the matter fields are suppressed as well and the effective theory on the E2-instanton provides the ADHM constraints for the massless E2-D6$_c$ open string modes.

3. Disc amplitudes

In order to evaluate the superpotential (2.2) for our locally engineered configuration of intersecting D6-branes, it essentially remains to compute the appearing disc diagrams with two fermionic respectively bosonic zero modes attached. Looking at the extended quiver diagram in figure 2, one realises that, due to the non-chirality of SQCD, numerous insertions of chains of matter fields Φ and $\widetilde{\Phi}$ are possible. However, in the field theory limit only the leading order term with the minimal number of Φ and $\widetilde{\Phi}$ insertions survives.

In this section we compute these relevant one- and two-point CFT disc amplitudes with two fermionic/bosonic zero modes in the background. To eventually compare our result to field theory, we first have to ensure that the string amplitudes and the vertex operators are normalised correctly. Here we will explicitly take care of factors of g_s, α' and v_c. As we will see at the end of the computation, additional (often convention dependent) numerical factors can always be absorbed into the definition of the dynamically generated scale Λ and are therefore not taken care of explicitly.

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branes	$\mathrm{D6}_c$	$\mathrm{D6}_f$	$\mathrm{D6}_c, \mathrm{D6}_f$	E2	$\mathrm{E2}, \mathrm{D6}_c$	$\mathrm{E2}, \mathrm{D6}_f$	$\mathrm{E2}, \mathrm{D6}_c, \mathrm{D6}_f$
C	$\frac{2\pi v_c}{g_s \ell_s^4}$	$\frac{2\pi v_f}{g_s \ell_s^4}$	$\frac{2\pi}{g_s \ell_s^4}$	$\frac{2\pi v_c}{g_s}$	$\frac{2\pi v_c}{g_s}$	$\frac{2\pi}{g_s}$	$\frac{2\pi}{g_s}$

Table 1: Disc normalisation factors C for the various combinations of possible boundaries.

3.1 Normalisation

In general, conformal field theory amplitudes carry a normalisation factor C, i.e.

$$\langle\langle \prod_{k=1}^{K} \Phi_k \rangle\rangle = C \int \frac{dz_1 \dots dz_K}{V_{1,2,K}} \, \langle V_{\Phi_1}(z_1) \, V_{\Phi_2}(z_2) \, \dots V_{\Phi_K}(z_K) \rangle \, , \tag{3.1}$$

where C depends on the boundaries of the disc. If there is only a single boundary brane Dp, then C is nothing else than the tension of the Dp-brane

$$C = \frac{\mu_p}{g_s} = \frac{2\pi}{g_s \, \ell_s^{p+1}} \, . \tag{3.2}$$

However, if the disc contains more boundaries the normalisation is given by the common sub-locus of all boundaries. For the relevant branes $\mathrm{D6}_c$, $\mathrm{D6}_f$ and E2 the resulting normalisation factors are shown in table 1.

Having fixed the normalisation factors of the disc, we have to take care of the normalisation of the vertex operators. These can be fixed by comparison with the standard normalisation in field theory. Let us first consider the vertex operators for the gauge boson in the (-1)-ghost picture

$$V_A^{(-1)}(z) = A^\mu \; e^{-\varphi(z)} \; \psi_\mu(z) \; e^{ip\cdot X(z)} \, . \tag{3.3}$$

In this normalisation the space-time field A^μ is dimensionless. The standard field theory normalisation is then obtained by scaling $A^\mu = (2\pi\alpha')^{\frac{1}{2}} A^\mu_{\mathrm{phys}}$. The Yang-Mills gauge coupling at the string scale is given by

$$\frac{1}{g_{\mathrm{YM}}^2} = \frac{1}{2} \, C_{\mathrm{D6}_c} (2\pi\alpha')^2. \tag{3.4}$$

In order not to overload the notation with too many scaling factors, in the following we first normalise the vertex operators such that the space-time fields do not carry any scaling dimension. Only in the very end we move to physical fields by introducing appropriate $(2\pi\alpha')^{\Delta/2}$ factors, where Δ denotes the four-dimensional scaling dimension of the field.

Next we have to normalise the matter fields localised on the intersection of the two D6-branes. We want the kinetic term to be normalised as in the work by Affleck, Dine and Seiberg

$$S_{\mathrm{matter}} = \int dx^4 \frac{1}{g_{\mathrm{YM}}^2} \, \partial_\mu \Phi \, \partial^\mu \overline{\Phi} \, . \tag{3.5}$$

Since now the disc normalisation does not contain the volume v_c, we have to put this factor in the normalisation of the vertex operators leading to

$$V_\Phi^{(-1)}(z) = \sqrt{v_c} \; \Phi \; e^{-\varphi(z)} \; \Sigma^{cf}_{1/2}(z) \; e^{ip\cdot X(z)} \, , \tag{3.6}$$

where $\Sigma_{1/2}^{cf}$ is an internal twist operator of conformal dimension $h = 1/2$.

As explained in section 2, for the instanton zero modes not to decouple in the field theory limit, their vertex operators must contain extra factors of $\sqrt{g_s}$. This leads to the following form of the vertex operator for the E2-D6$_c$ bosonic zero modes b_c

$$V_{b_c}^{(-1)}(z) = \sqrt{\frac{g_s}{2\pi v_c}}\; b_c\; e^{-\varphi(z)} \prod_{\mu=0}^{3} \sigma^{\mu}_{\frac{1}{16}}(z) \prod_{\nu=0}^{3} s^{\nu}_{\frac{1}{16}}(z) \;, \tag{3.7}$$

where the $\sigma^{\mu}_{\frac{1}{16}}(z)$ denote the bosonic twist fields of conformal dimension $h = 1/16$ and $s^{\nu}_{\frac{1}{16}}(z)$ the fermionic ones of the same conformal dimension. Similarly, for the fermionic instanton zero modes in this sector we write

$$V_{\beta_c}^{(-1/2)}(z) = \sqrt{\frac{g_s}{2\pi v_c}}\; \beta_c\; e^{-\frac{\varphi(z)}{2}} \prod_{\mu=0}^{3} \sigma^{\mu}_{\frac{1}{16}}(z)\; \Sigma^{c}_{3/8}(z) \;. \tag{3.8}$$

Taking into account the different disc normalisations for the charged matter fermionic zero modes, the vertex operator is

$$V_{\lambda_f}^{(-1/2)}(z) = \sqrt{\frac{g_s}{2\pi}}\; \lambda_f\; e^{-\frac{\varphi(z)}{2}} \prod_{\mu=0}^{3} \sigma^{\mu}_{\frac{1}{16}}(z)\; \Sigma^{f}_{3/8}(z) \;. \tag{3.9}$$

In the fermionic vertex operators, $\Sigma_{3/8}$ denote appropriate Ramond fields, whose form depends on the concrete CFT describing of the internal Calabi-Yau manifold.

3.2 Three-point amplitudes

As we mentioned earlier, in the field theory limit (2.32) the computation of the single instanton generated superpotential (2.2) simplifies considerably. In particular, for our setup shown in figure 2 we are left with only two disc diagrams with insertion of two fermionic zero modes from the set $\beta_c, \widetilde{\beta}_c, \lambda_f, \widetilde{\lambda}_f$. See for instance [44 – 48] for the conformal field theory computation of disc amplitudes for intersecting D-brane models. The appropriate combination of these zero modes can be obtained by looking at the extended quiver diagram in figure 2. However, because of $U(1)$ charge cancellation on the world-sheet, the fermionic zero modes couple to anti-holomorphic matter fields. The explicit form of the first disc correlator is

$$\langle\langle\, \overline{\widetilde{\Phi}}_{cf}\, \rangle\rangle^{\rm disc}_{\beta_c \widetilde{\lambda}_f} = \frac{2\pi}{g_s} \int \frac{dz_1\, dz_2\, dz_3}{V_{123}}\; \langle V_{\beta_c}^{(-1/2)}(z_1)\, V_{\overline{\widetilde{\Phi}}_{cf}}^{(-1)}(z_2)\, V_{\widetilde{\lambda}_f}^{(-1/2)}(z_3)\rangle \;. \tag{3.10}$$

The second disc correlator is $\langle\langle\, \overline{\Phi}_{fc}\, \rangle\rangle^{\rm disc}_{\lambda_f \widetilde{\beta}_c}$ and has a form analogous to the first one.

The evaluation of these two disc correlators is easily achieved using the following three-point functions

$$\begin{aligned}
\langle \sigma^{\mu}_{\frac{1}{16}}(z_1)\, \sigma^{\nu}_{\frac{1}{16}}(z_3)\rangle &= \frac{\delta^{\mu\nu}}{(z_1 - z_3)^{\frac{1}{8}}} \;, \\
\langle e^{-\frac{\varphi(z_1)}{2}}\, e^{-\varphi(z_2)}\, e^{-\frac{\varphi(z_3)}{2}}\rangle &= \frac{1}{(z_1 - z_2)^{\frac{1}{2}}\, (z_1 - z_3)^{\frac{1}{4}}\, (z_2 - z_3)^{\frac{1}{2}}} \;, \\
\langle \Sigma^{c}_{\frac{3}{8}}(z_1)\, \overline{\widetilde{\Sigma}}^{cf}_{\frac{1}{2}}(z_2)\, \Sigma^{f}_{\frac{3}{8}}(z_3)\rangle &= \frac{1}{(z_1 - z_2)^{\frac{1}{2}}\, (z_1 - z_3)^{\frac{1}{4}}\, (z_2 - z_3)^{\frac{1}{2}}} \;,
\end{aligned} \tag{3.11}$$

and we directly obtain the effective term in the instanton action

$$\mathcal{L}_{\text{ferm}} = \beta_c\, \overline{\widetilde{\Phi}}_{cf}\, \widetilde{\lambda}_f + \lambda_f\, \overline{\Phi}_{fc} \widetilde{\beta}_c\ . \tag{3.12}$$

Note that this is the same Lagrangian as appears in the field theory calculation [15].

3.3 Four-point amplitude

In the field theory limit we essentially have to compute one disc amplitude with insertion of two bosonic zero modes. Looking again at the extended quiver in figure 2, we are led to insert the bosonic zero modes $\{b, \overline{b}\}$ and $\{\overline{\widetilde{b}}, \widetilde{b}\}$.[7] Because of $U(1)$ charge cancellation on the world-sheet, the matter fields can only be paired as $\Phi\,\overline{\Phi}$ and as $\overline{\widetilde{\Phi}}\,\widetilde{\Phi}$. The explicit form of the CFT four-point function is the following

$$\langle\langle\, \Phi_{cf} \overline{\Phi}_{fc'}\, \rangle\rangle^{\text{disc}}_{b_c\, \overline{b}_{c'}} = \frac{2\pi}{g_s} \int \frac{dz_1\, dz_2\, dz_3\, dz_4}{V_{124}}\, \langle V^{(-1)}_{b_c}(z_1)\, V^{(0)}_{\Phi_{cf}}(z_2)\, V^{(0)}_{\overline{\Phi}_{fc'}}(z_3)\, V^{(-1)}_{\overline{b}_{c'}}(z_4) \rangle\ , \tag{3.13}$$

and similarly for $\{\overline{\widetilde{b}}, \widetilde{b}\}$ and $\overline{\widetilde{\Phi}}\,\widetilde{\Phi}$. The vertex operators for the bosons of the matter superfields in the (0)-ghost picture are given by

$$V^{(0)}_{\Phi}(z) = \sqrt{v_c}\, \Phi\ \left(\Sigma_1(z) + i(p\cdot\psi)\, \Sigma_{1/2}(z)\right)\, e^{ip\cdot X(z)}\ , \tag{3.14}$$

where $\Sigma_1(z) = \{G_{-1/2}, \Sigma_{1/2}(z)\}$ is an internal operator of conformal dimension $h = 1$. The four-point function in the ghost sector and the internal sector simply reduce to two-point functions

$$\begin{aligned}
\langle e^{-\varphi(z_1)}\, e^{-\varphi(z_4)} \rangle &= \frac{1}{(z_1 - z_4)}\ , & \langle s^{\mu}_{\frac{1}{16}}(z_1)\, s^{\nu}_{\frac{1}{16}}(z_4) \rangle &= \frac{\delta^{\mu\nu}}{(z_1 - z_4)^{\frac{1}{8}}}\ ,\\
\langle \Sigma_{1/2}(z_2)\, \overline{\Sigma}_{1/2}(z_3) \rangle &= \frac{1}{(z_2 - z_3)}\ , & \langle \Sigma_1(z_2)\, \overline{\Sigma}_1(z_3) \rangle &= \frac{1}{(z_2 - z_3)^2}\ .
\end{aligned} \tag{3.15}$$

What remains to be computed are the four-point functions

$$\langle \sigma^{\mu}_{\frac{1}{16}}(\infty)\, e^{ip_\mu X^\mu(z_2)}\, e^{-ip_\mu X^\mu(z_3)}\, \sigma^{\mu}_{\frac{1}{16}}(0) \rangle \tag{3.16}$$

and

$$\langle s^{\mu}_{\frac{1}{16}}(\infty)\, \psi_\mu(z_2)\, \psi_\mu(z_3)\, s^{\mu}_{\frac{1}{16}}(0) \rangle \tag{3.17}$$

(no sum over μ). However, these functions are nothing else than the respective two-point functions in the $\mathbb{Z}_2$ twisted sector, which can be obtained from

$$\langle X^\mu(z)\, X^\nu(w) \rangle_T = -\eta^{\mu\nu}\, \ln\left[\frac{z - w}{(\sqrt{z} + \sqrt{w})^2}\right] \tag{3.18}$$

[7] Naively, it is also possible to pair the zero modes as $\{b, \widetilde{b}\}$ and $\{\overline{\widetilde{b}}, \overline{b}\}$. However, let us for the moment go back to the notation $b_{\alpha,c}$ from the beginning and attach labels α also to the spin fields $s^{\alpha}_{\frac{1}{16}}$ of equation (3.15). Equation (A.19) in [27] tells us that $\langle s^{\alpha}_{\frac{1}{16}}(z) s^{\beta}_{\frac{1}{16}}(w) \rangle \sim -\frac{\epsilon^{\alpha\beta}}{(z-w)^{1/2}}$ and therefore only the insertion of $\{b_{1,c}, \overline{b}_{2,c}\}$ or $\{b_{2,c}, \overline{b}_{1,c}\}$ into the disc amplitude gives a non-vanishing result. Converting this back into our notation, this is precisely the choice we mentioned.

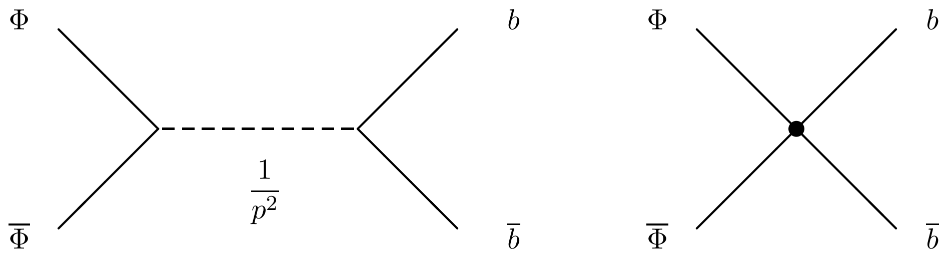

Figure 3: Four-point massless string exchange and four-point vertex interaction.

and

$$\langle \psi^\mu(z)\,\psi^\nu(w)\rangle_T = \frac{\eta^{\mu\nu}}{2}\,\frac{\sqrt{\frac{z}{w}}+\sqrt{\frac{w}{z}}}{z-w}\,. \tag{3.19}$$

Combining all these terms and substituting $x^2 = z_3$, the four-point function (3.13) can be expressed as

$$\langle\langle\,\Phi_{cf}\overline{\Phi}_{fc'}\,\rangle\rangle^{\rm disc}_{b_c\,\overline{b}_{c'}} = b_c\,\Phi_{cf}\,\overline{\Phi}_{fc'}\,\overline{b}_{c'}\int_0^1 dx\,\frac{(1+x)^{p^2-2}}{(1-x)^{p^2+2}}\,\big(2x+p^2\,(x^2+1)\big)\,. \tag{3.20}$$

The integral is elementary and one finally obtains

$$\langle\langle\,\Phi\overline{\Phi}\,\rangle\rangle^{\rm disc}_{b_c\,\overline{b}_c} = b_c\,\Phi\,\overline{\Phi}\,\overline{b}_c\left[2^{p^2-1}\lim_{x\to1}\,(1-x)^{-p^2-1}-\frac{1}{2}\right]. \tag{3.21}$$

This four-point string amplitude corresponds to the two field theory diagrams shown in figure 3. The first term, corresponding to the first diagram, is divergent for $p^2\to0$, i.e. on-shell, and reflects the singularity due to the exchange of massless states in the four-point amplitude. The second term corresponds to the second diagram and is momentum-independent. Therefore, it should be interpreted as the local four-point interaction.

In summary, taking also into account the insertion of the bosonic zero modes $\{\overline{\widetilde{b}},\widetilde{b}\}$ and the matter field pairings $\overline{\widetilde{\Phi}}\,\widetilde{\Phi}$, we obtain the effective term in the instanton action originating from bosonic zero modes as

$$\mathcal{L}_{\rm bos} = \tfrac{1}{2}\,b_c\left(\Phi_{cf}\,\overline{\Phi}_{fc'}+\overline{\widetilde{\Phi}}_{cf}\,\widetilde{\Phi}_{fc'}\right)\overline{b}_{c'}+\tfrac{1}{2}\,\overline{\widetilde{b}}_c\left(\Phi_{cf}\,\overline{\Phi}_{fc'}+\overline{\widetilde{\Phi}}_{cf}\,\widetilde{\Phi}_{fc'}\right)\widetilde{b}_{c'}\,. \tag{3.22}$$

This again agrees with the field theory Lagrangian [15].

4. The Affleck-Dine-Seiberg superpotential

Having engineered a local intersecting D6-brane model giving rise to $SU(N_c)$ super Yang-Mills theory with N_f non-chiral flavours and having determined the leading order disc diagrams, we now come to the evaluation of the zero mode integrals in (2.2) to see whether one can really recover the Affleck-Dine-Seiberg (ADS) superpotential

$$W = \frac{\Lambda^{3N_c - N_f}}{\det(M_{ff'})} \,. \tag{4.1}$$

Note that here the matter fields carry their canonical scaling dimension $\Delta = 1$. The dynamically generated scale Λ is defined as

$$\left(\frac{\Lambda}{\mu}\right)^{3N_c - N_f} = \exp\left(-\frac{8\pi^2}{g_c^2(\mu)}\right) \,. \tag{4.2}$$

Performing the corresponding single E2-instanton computation in the local D-brane model, we are aiming to explicitly derive the ADS superpotential in the field theory limit. Since we think that our methods are applicable to more general instanton computations, we provide the actual evaluation of the zero mode integrals in quite some detail.

4.1 Fermionic zero mode integration

In this subsection we compute to lowest order in α' the contribution of the fermionic zero modes to the E2-instanton generated superpotential (2.2). The relevant disc diagrams have been computed in section 3.2. However, since we will perform an integration over instanton zero modes, we should replace the matter fields Φ_{cf} by their VEVs $\langle \Phi_{cf} \rangle$ and similarly for $\widetilde{\Phi}_{fc}$ and their conjugates. Furthermore, we have to satisfy the D-term constraint for the matter fields to ensure the supersymmetry of our setup so we can apply formula (2.2). Specifically, the D-term constraint implies

$$\langle \Phi_{cf} \rangle = \langle \overline{\widetilde{\Phi}}_{cf} \rangle \qquad \text{and} \qquad \langle \widetilde{\Phi}_{cf} \rangle = \langle \overline{\Phi}_{cf} \rangle \,. \tag{4.3}$$

Using these relations, one can rewrite (3.12) to obtain the relevant three-point couplings

$$\beta_c \, \langle \Phi_{cf} \rangle \, \widetilde{\lambda}_f + \lambda_f \, \langle \widetilde{\Phi}_{fc} \rangle \widetilde{\beta}_c \tag{4.4}$$

where $c = 1, \ldots, N_c$ and $f = 1, \ldots, N_f$. The sum over repeated indices is understood and in the following we will omit the VEV brackets for ease of notation.

The next step is to implement the fermionic ADHM constraints (2.22) in terms of delta-functions. For Grassmann-variables they read

$$\delta\left(\widetilde{\beta}_{c_1} \, b_{c_1} + \beta_{c_2} \, \widetilde{b}_{c_2}\right) \, \delta\left(\widetilde{\beta}_{c_3} \, \overline{\widetilde{b}}_{c_3} - \beta_{c_4} \, \bar{b}_{c_4}\right) = \left(\widetilde{\beta}_{c_1} \, b_{c_1} + \beta_{c_2} \, \widetilde{b}_{c_2}\right) \left(\widetilde{\beta}_{c_3} \, \overline{\widetilde{b}}_{c_3} - \beta_{c_4} \, \bar{b}_{c_4}\right) . \tag{4.5}$$

Taking also into account the fermionic part of the integration measure (2.24), we arrive at the following expression for the fermionic contribution to the superpotential

$$\begin{aligned} I_{\text{ferm}}(\Phi, \widetilde{\Phi}, b, \widetilde{b}) = \int \prod_{c=1}^{N_c} d\beta_c d\widetilde{\beta}_c \prod_{f=1}^{N_f} d\lambda_f d\widetilde{\lambda}_f \, & \left(\widetilde{\beta}_{c_1} b_{c_1} \overline{\widetilde{b}}_{c_2} \widetilde{\beta}_{c_2} - \widetilde{\beta}_{c_1} b_{c_1} \bar{b}_{c_2} \beta_{c_2} \right. \\ & \left. + \beta_{c_1} \widetilde{b}_{c_1} \overline{\widetilde{b}}_{c_2} \widetilde{\beta}_{c_2} - \beta_{c_1} \widetilde{b}_{c_1} \bar{b}_{c_2} \beta_{c_2} \right) \, \exp\left(\beta_c \Phi_{cf} \widetilde{\lambda}_f + \lambda_f \widetilde{\Phi}_{fc} \widetilde{\beta}_c\right) \,. \end{aligned} \tag{4.6}$$

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Note the dependence not only on the matter fields but also on the bosonic zero modes $b, \widetilde{b}$. The fermionic modes β and $\widetilde{\beta}$ in front of the exponent determine which part in the series expansion is picked out by the integral. For instance, in the case of a prefactor $\widetilde{\beta}\beta$ only $(\beta_c \Phi_{cf} \widetilde{\lambda}_f)^{N_c-1}$ and $(\lambda_f \widetilde{\Phi}_{fc} \widetilde{\beta}_c)^{N_c-1}$ survive the $d\beta$ integration. This implies, however, that the integral (4.6) can only be non-zero if and only if

$$N_f = N_c - 1 \; . \tag{4.7}$$

One can check that for prefactors $\beta\beta$ and $\widetilde{\beta}\,\widetilde{\beta}$ the complete fermionic integral vanishes, since the number of modes λ is equal the number of modes $\widetilde{\lambda}$. Therefore, we recovered nicely the constraint of Affleck, Dine and Seiberg for the instanton generated ADS superpotential.

By differentiating under the integral sign, and using some of the formulae of appendix A.1, one finds that the final result for the fermionic zero mode integration is

$$I_{\text{ferm}}(\Phi, \widetilde{\Phi}, b, \widetilde{b}) = \sum_{p,q=1}^{N_c} (-1)^{p+q} \left(b_p \overline{b}_q + \overline{\widetilde{b}}_p \widetilde{b}_q \right) \det\left[\Phi \widetilde{\Phi}\big|_{q,p} \right] \; . \tag{4.8}$$

Here, and in the following, Φ is the $N_c \times (N_c - 1)$ matrix with elements $\Phi_{c,f}$ and similarly $\widetilde{\Phi}$ is an $(N_c - 1) \times N_c$ matrix. The symbol $A|_{q,p}$ denotes the matrix obtained from A by deleting the q'th row and p'th column. The (q,p)'th element of a matrix A will be denoted by $A_{q,p}$.

4.2 Bosonic zero mode integration

Let us now turn to the evaluation of the bosonic zero mode integrals in (2.24). The relevant disc diagrams have been computed in section 3.3. Following the same reasoning as for the three-point amplitudes, we replace the matter fields by their VEVs and use the D-term constraint (4.3) to arrive at the four-point couplings

$$b_c \, \langle \Phi_{cf} \rangle \, \langle \widetilde{\Phi}_{fc'} \rangle \, \overline{b}_{c'} + \overline{\widetilde{b}}_c \, \langle \Phi_{cf} \rangle \, \langle \widetilde{\Phi}_{fc'} \rangle \, \widetilde{b}_{c'} \; . \tag{4.9}$$

Note that in the following we will again omit the VEV brackets for ease of notation. The ADHM constraints for the bosonic moduli space, or equivalently the D- and F-term constraints for the E2, are given in equation (2.21). Again, these have to be implemented as delta-functions in the integration measure (2.24). The explicit form of the bosonic part reads

$$\begin{aligned} &\int \prod_{c=1}^{N_c} db_c \, d\overline{b}_c \, d\widetilde{b}_c \, d\overline{\widetilde{b}}_c \; \delta\left(\overline{b}_c \overline{\widetilde{b}}_c + \widetilde{b}_c b_c \right) \delta\left(i\overline{b}_c \overline{\widetilde{b}}_c - i\widetilde{b}_c b_c \right) \delta\left(\overline{b}_c b_c - \widetilde{b}_c \overline{\widetilde{b}}_c \right) \\ &\quad = \int \prod_{c=1}^{N_c} db_c \, d\overline{b}_c \, d\widetilde{b}_c \, d\overline{\widetilde{b}}_c \int dk_1 \exp\left(ik_1 (\overline{b}_c \overline{\widetilde{b}}_c + \widetilde{b}_c b_c) \right) \\ &\qquad\qquad \times \int dk_2 \exp\left(-k_2 (\overline{b}_c \overline{\widetilde{b}}_c - \widetilde{b}_c b_c) \right) \int dk_3 \exp\left(ik_3 (\overline{b}_c b_c - \widetilde{b}_c \overline{\widetilde{b}}_c) \right) \; . \end{aligned} \tag{4.10}$$

Finally, we combine (4.9), (4.10) and the bosonic fields from the fermionic integration (4.8) into the bosonic integral

$$\int d^3k \int \prod_{c=1}^{N_c} db_c\, d\bar{\tilde{b}}_c\, d\bar{b}_c\, d\tilde{b}_c \left(b_p\bar{b}_q + \bar{\tilde{b}}_p\tilde{b}_q\right) \exp\left(-\begin{bmatrix} b \\ \bar{\tilde{b}} \end{bmatrix}^T \begin{bmatrix} M_{(1)} & M_{(2)} \\ M_{(3)} & M_{(4)} \end{bmatrix} \begin{bmatrix} \bar{b} \\ \tilde{b} \end{bmatrix}\right), \tag{4.11}$$

where the b's are vectors with the N_c entries b_c and the M's are $N_c \times N_c$ matrices of the following form

$$\begin{aligned} M_{(1)} &= -\Phi\widetilde{\Phi} + \epsilon\,\mathbb{1} - ik_3\,\mathbb{1}\,, & M_{(2)} &= -ik_1\,\mathbb{1} - k_2\,\mathbb{1}\,, \\ M_{(3)} &= -ik_1\,\mathbb{1} + k_2\,\mathbb{1}\,, & M_{(4)} &= -\Phi\widetilde{\Phi} + \epsilon\,\mathbb{1} + ik_3\,\mathbb{1}\,. \end{aligned} \tag{4.12}$$

Here were added an infinitesimal parameter $\epsilon > 0$ in order to regularise the complex Gaussian integrals (4.11). At the end of the computation we will take the $\epsilon \to 0$ limit. Note that bosonic fields in front of the exponential can be written as

$$b_p\bar{b}_q + \bar{\tilde{b}}_p\tilde{b}_q = -\frac{\partial}{\partial M_{(1)p,q}} - \frac{\partial}{\partial M_{(4)p,q}}\,, \tag{4.13}$$

where for instance the first derivative is with respect to the (p,q)'th element of the matrix $M_{(1)}$. Then one can perform the Gaussian integrals in (4.11) to obtain

$$\int d^3k \left(-\frac{\partial}{\partial M_{(1)p,q}} - \frac{\partial}{\partial M_{(4)p,q}}\right)\ \det\left(\begin{bmatrix} M_{(1)} & M_{(2)} \\ M_{(3)} & M_{(4)} \end{bmatrix}\right)^{-1} \tag{4.14}$$

$$= \int d^3k \left(\left(M^{-1}_{(1)}\right)_{q,p} + \left(M^{-1}_{(4)}\right)_{q,p}\right)\ \det\left(\begin{bmatrix} M_{(1)} & M_{(2)} \\ M_{(3)} & M_{(4)} \end{bmatrix}\right)^{-1} \tag{4.15}$$

$$= -2\int d^3k \sum_{r=1}^{N_c} \frac{\left(\Phi\widetilde{\Phi} - \epsilon\mathbb{1}\right)_{q,r}\left(\left(\Phi\widetilde{\Phi} - \epsilon\mathbb{1}\right)^2 + k^2\mathbb{1}\right)^{-1}_{r,p}}{\det\left[\left(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\right)^2 + k^2\,\mathbb{1}\right]}\,. \tag{4.16}$$

From line (4.14) to (4.15) we used formula (A.1) from the appendix. From (4.15) to line (4.16) we used (A.2) and (A.3). Note that $M^{-1}_{(1)}$ stands for the upper-left block of the block-matrix M^{-1}, i.e. it is not the inverse of $M_{(1)}$.

At this point let us summarise the results of the fermionic and bosonic zero mode integration so far. Combining equations (4.8) and (4.16) and going to spherical coordinates we find the expression

$$\begin{aligned} I_{\text{bos}}(\Phi,\widetilde{\Phi}) = \int d^4x\, d^2\theta \int_0^\infty dk\, k^2 \\ \times \sum_{p,q,r=1}^{N_c} (-1)^{p+q}\, \frac{\det\left[\Phi\widetilde{\Phi}\big|_{q,p}\right]\left(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\right)_{q,r}\left(\left(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\right)^2 + k^2\,\mathbb{1}\right)^{-1}_{r,p}}{\det\left[\left(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\right)^2 + k^2\,\mathbb{1}\right]}\,. \end{aligned} \tag{4.17}$$

Note that so far still the combination $\Phi\widetilde{\Phi}$ occurs, whereas eventually we have to get the flavour matrix $\widetilde{\Phi}\Phi$. In (4.17) there is a part which can be simplified using equations (A.4) and (A.5)

$$\sum_{q=1}^{N_c} (-1)^{p+q} \det\big[\Phi\widetilde{\Phi}\big|_{q,p}\big] \; \Phi\widetilde{\Phi}_{q,r} = \det\big[\Phi\widetilde{\Phi}\big]\, \delta_{p,r} = 0 \,. \tag{4.18}$$

The remaining part can be rewritten using equations (A.6) and (A.7) from the appendix

$$\begin{aligned}
&\epsilon \int_0^\infty dk\; k^2 \sum_{p,q=1}^{N_c} (-1)^{2p+2q} \frac{\det\big[\Phi\widetilde{\Phi}\big|_{q,p}\big] \;\det\Big[\Big(\big(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\big)^2 + k^2\,\mathbb{1}\Big)\Big|_{p,q}\Big]}{\det\Big[\big(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\big)^2 + k^2\,\mathbb{1}\Big]^2} \\
&\qquad = \epsilon \int_0^\infty dk\; k^2 \frac{\sum_{p=1}^{N_c} \;\det\Big[\Big(\Phi\widetilde{\Phi}\big(\big(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\big)^2 + k^2\,\mathbb{1}\big)\Big)\Big|_{p,p}\Big]}{\det\Big[\big(\Phi\widetilde{\Phi} - \epsilon\,\mathbb{1}\big)^2 + k^2\,\mathbb{1}\Big]^2} \,.
\end{aligned} \tag{4.19}$$

This expression can be simplified using the results of appendix A.2 for the numerator and of appendix A.3 for the denominator

$$\begin{aligned}
=&\epsilon \int_0^\infty dk\; k^2 \frac{\det\Big[\widetilde{\Phi}\Phi\big(\big(\widetilde{\Phi}\Phi - \epsilon\,\mathbb{1}\big)^2 + k^2\,\mathbb{1}\big)\Big]}{\big(ik-\epsilon\big)^2\big(ik+\epsilon\big)^2 \det\big[\widetilde{\Phi}\Phi - \epsilon\,\mathbb{1} + ik\,\mathbb{1}\big]^2 \det\big[\widetilde{\Phi}\Phi - \epsilon\,\mathbb{1} - ik\,\mathbb{1}\big]^2} \\
=&\epsilon \int_0^\infty dk\; k^2 \frac{\det\big[\widetilde{\Phi}\Phi\big]}{\big(ik-\epsilon\big)^2\big(ik+\epsilon\big)^2 \det\big[\widetilde{\Phi}\Phi - \epsilon\,\mathbb{1} + ik\,\mathbb{1}\big] \det\big[\widetilde{\Phi}\Phi - \epsilon\,\mathbb{1} - ik\,\mathbb{1}\big]} \,.
\end{aligned} \tag{4.20}$$

Finally, let us denote the eigenvalues of the matrix $\widetilde{\Phi}\Phi$ as σ_j with $j = 1, \ldots, N_c - 1$. Then one obtains for equation (4.17) the following expression

$$\begin{aligned}
I_{\mathrm{bos}}(\Phi, \widetilde{\Phi}) = &\int d^4x\, d^2\theta \;\det\big[\widetilde{\Phi}\Phi\big] \\
&\times \epsilon \int_0^\infty dk \; \frac{k^2}{\big(ik-\epsilon\big)^2\big(ik+\epsilon\big)^2 \prod_{j=1}^{N_c-1}\big(\sigma_j - \epsilon + ik\big)\big(\sigma_j - \epsilon - ik\big)} \,.
\end{aligned} \tag{4.21}$$

Let us denote the integrand in the last line as $f(k)$. Due to the symmetry $k \to -k$ of the integral and the vanishing of the integrand at infinity, one can rewrite (4.21) as a contour integral

$$I_{\mathrm{bos}}(\Phi, \widetilde{\Phi}) = \int d^4x\, d^2\theta \;\det\big[\widetilde{\Phi}\Phi\big] \,\frac{\epsilon}{2\pi i} \oint f(k) \tag{4.22}$$

over the upper half-plane, for instance. The contour integral can be evaluated by virtue of the residue theorem

$$\frac{\epsilon}{2\pi i} \oint f(k) = \epsilon\,\mathrm{Res}\left[f(k), k = i\epsilon\right] + \epsilon \sum_{j=1}^{N_c-1} \mathrm{Res}\left[f(k), k = i(\sigma_j - \epsilon)\right] \,. \tag{4.23}$$

Generically, all eigenvalues of the matrix $\widetilde{\Phi}\Phi$ are nonzero and one finds for the first term

$$\epsilon\,\mathrm{Res}\left[f(k), k = i\epsilon\right] = -\frac{i}{4}\, \frac{1}{\prod_{j=1}^{N_c-1}\big(\sigma_j - 2\epsilon\big)\sigma_j} \left(1 + \sum_{j=1}^{N_c-1} \frac{2\epsilon^2}{\big(\sigma_j - 2\epsilon\big)\sigma_j}\right) \,. \tag{4.24}$$

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For the calculation of all the other residues in (4.23) one can assume that $|\sigma_j| > \epsilon$. Therefore, the limit $\epsilon \to 0$ can be performed before evaluating the residue and so all these terms vanish. Taking the limit $\epsilon \to 0$ in expression (4.24), the result simplifies drastically and one obtains

$$\lim_{\epsilon \to 0} \text{Res}\left[f(k), k = i\epsilon\right] = -\frac{i}{4} \frac{1}{\prod_{j=1}^{N_c-1} \sigma_j^2} = -\frac{i}{4} \frac{1}{\det\left[\widetilde{\Phi}\Phi\right]^2} \,. \tag{4.25}$$

Using this result for the contour integration, we arrive at our final result for the bosonic zero mode integration

$$I_{\text{bos}}(\Phi, \widetilde{\Phi}) = \int d^4x\, d^2\theta\, \frac{1}{\det\left[\widetilde{\Phi}\Phi\right]} \,. \tag{4.26}$$

The last step to obtain the E2-instanton generated superpotential (2.2) is to include the contribution of $\exp(\langle 1 \rangle^{\text{disc}})$

$$S_W = \frac{2\pi^2}{\ell_s^3} \int d^4x\, d^2\theta\, \frac{1}{\det\left[\widetilde{\Phi}\Phi\right]} \exp\left(-\frac{8\pi^2}{g_c^2(M_s)}\right) \tag{4.27}$$

with $M_s = (\alpha')^{-1/2}$. Transforming to canonically normalised matter fields, $\Phi = (2\pi\alpha')^{1/2}\, \Phi_{\text{phys}}$, we get

$$S_W = N \int d^4x\, d^2\theta\, \frac{M_s^{2N_f+3}}{\det\left[\widetilde{\Phi}_{\text{phys}}\Phi_{\text{phys}}\right]} \exp\left(-\frac{8\pi^2}{g_c^2(M_s)}\right) \,, \tag{4.28}$$

where we have collected all numerical factors appearing during the computation into the constant N.

Using (4.2) for the dynamically generated scale Λ, while absorbing the numerical factor into Λ, gives precisely the ADS superpotential

$$S_W = \int d^4x\, d^2\theta\, \frac{\Lambda^{3N_c - N_f}}{\det[M_{ff'}]} \,. \tag{4.29}$$

Therefore, we have derived the ADS superpotential in the field theory limit from a D-brane instanton. As mentioned, in string theory there will be numerous corrections to this simple expression. All the massive string states will appear in the one-loop determinant and there will be an infinite series of world-sheet instanton corrections to the disc amplitudes. Moreover, multiple matter field insertions along the boundary of the disc are possible and give higher α' corrections to the ADS superpotential. Note that these higher order terms will break some of the global symmetries, like R-symmetry, present in the field theory limit. Therefore, one warning is in order at this point:

> Whenever one simply extends, for instance, string flux superpotentials by non-perturbative superpotentials from pure field theory, like the ADS superpotential, one has to ensure that one always remains in the regime of validity of the field theory limit of string theory, i.e. at large transversal radii and small VEVs for the matter fields, i.e. $\langle \Phi \rangle \ll M_s$.

We would also like to add a few methodological remarks concerning our derivation of the ADS superpotential. Recall that Affleck, Dine and Seiberg inferred the form of their superpotential solely from considerations of symmetry and then calculated that it is in fact generated by instantons. Therefore, whenever one has constructed an SQCD theory with $N_f = N_c - 1$, regardless of its origin, it is a priori clear that it should feature an ADS superpotential. In this regard, our stringy calculation of the ADS superpotential can also be considered a benchmark for the instanton calculus of [21], showing in a non-trivial way that the prescription given there leads to the correct, known result in our setup.

5. Superpotentials for the other classical gauge groups

The natural question now is whether our results can be generalised to other known cases of dynamically generated superpotentials for $\mathcal{N} = 1$ super Yang-Mills theories. Indeed, field theories with $USp(2N_c)$[8] and $SO(N_c)$ gauge group with N_f flavours have been discussed in the literature.

These gauge groups are realised on branes lying right on top of the orientifold plane. The first question to address is how the instanton zero mode structure is modified. So imagine starting with a stack of N_c branes and an instanton wrapping the same three-cycle on the internal manifold. As outlined in section 2 there are N_c fermionic zero modes β_c and $2N_c$ bosonic zero modes b_c, $\widetilde{\bar{b}}_c$ from strings starting on the instanton and ending on the D6-brane as well as N_c fermionic zero modes $\widetilde{\beta}_c$ and $2N_c$ bosonic zero modes $\bar{b}_c$, $\widetilde{b}_c$ from strings starting on the D6-brane and ending on the instanton. Upon adding an orientifold plane wrapping the same cycle as the branes (and the instanton) only one linear combination of β_c and $\widetilde{\beta}_c$, b_c and $\widetilde{b}_c$ as well as $\bar{b}_c$ and $\widetilde{\bar{b}}_c$ is kept. This effectively amounts to identifying β_c with $-\widetilde{\beta}_c$, b_c with $-\widetilde{b}_c$ and $\bar{b}_c$ with $-\widetilde{\bar{b}}_c$.

5.1 Symplectic gauge group

Consider now a setup where the orientifold is such that the gauge group on the $2N_c$ branes lying on top of it is $USp(2N_c)$. Due to the Dirichlet boundary conditions for the strings attached to the E2 in the four non-compact dimensions, the gauge symmetry on the instanton is $SO(1)$.[9] The gauge symmetry on E2 being trivial, we do not expect D-term and F-term constraints for E2. And indeed, as can be found for instance in [49], there are neither bosonic nor fermionic ADHM constraints. The stringy interpretation of this is the following: At least one field in each interaction term that realises the ADHM constraints in [27] is odd under the orientifold projection (see also [41]). This is true in particular for the instanton zero modes $\overline{\theta}_i$ and thus the corresponding ADHM constraints are absent. As explained above there are now $2N_c$ fermionic zero modes β_c and $4N_c$ bosonic zero modes b_c, $\bar{b}_c$. All zero modes β_c, b_c and $\bar{b}_c$ transform in the fundamental representation of $USp(2N_c)$.

Furthermore, we denote the invariant matter strings from the colour brane to the flavour brane by Φ, those from the flavour to the colour brane by $\widetilde{\Phi}$ and the fermionic zero

[8] Our convention is such that $USp(2) \cong SU(2)$.

[9] Note that the auxiliary group for field theory instantons in symplectic (and orthogonal, see next section) gauge groups [49] is nicely reproduced in string theory.

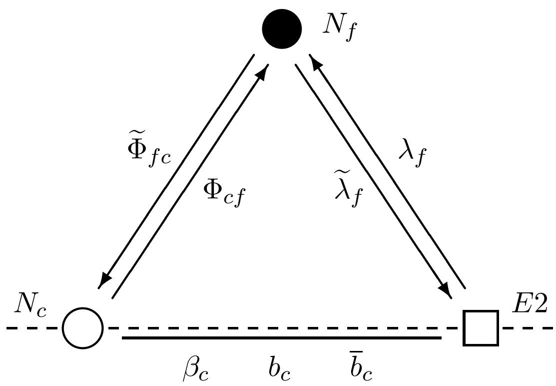

Figure 4: Brane configuration for an instanton in an $USp(2N_c)$ gauge theory. The dashed line represents the orientifold plane.

modes from strings between the flavour brane and the instanton by λ_f and $\widetilde{\lambda}_f$. All these conventions are shown in the extended quiver diagram in figure 4.

In analogy to the $U(1)$ instanton case, to lowest order in α' the possible disc amplitudes with insertion of fermionic zero modes are

$$\beta_c\,\overline{\widetilde{\Phi}}_{cf}\,\widetilde{\lambda}_f + \lambda_f\,\overline{\Phi}_{fc}\,\widetilde{\beta}_c = \beta_c\,\overline{\widetilde{\Phi}}_{cf}\,\widetilde{\lambda}_f + \lambda_f\,\overline{\Phi}_{fc}\,(-\beta)_c = \beta_c\left[\overline{\widetilde{\Phi}}\,,\,\overline{\Phi}^T\right]_{cf}\begin{bmatrix}\widetilde{\lambda}\\ \lambda\end{bmatrix}_f \tag{5.1}$$

where we again omitted the VEV brackets for the matter fields. Note that $[\,\overline{\widetilde{\Phi}}\,,\,\overline{\Phi}^T\,]_{cf}$ is a $2N_c \times 2N_f$ matrix since the fields $\overline{\widetilde{\Phi}}_{cf}$ and $\overline{\Phi}_{fc}$ both transform in bifundamental representations of $USp(2N_c) \times U(N_f)$. Since the ADHM constraints are absent, the integration over fermionic zero modes can easily be performed

$$\int \prod_{c=1}^{2N_c} d\beta_c \prod_{f=1}^{N_f} d\lambda_f d\widetilde{\lambda}_f\ \exp\left(\beta_c\left[\overline{\widetilde{\Phi}}\,,\,\overline{\Phi}^T\right]_{cf}\begin{bmatrix}\widetilde{\lambda}\\ \lambda\end{bmatrix}_f\right) = \det\left[\overline{\widetilde{\Phi}}\,,\,\overline{\Phi}^T\right]\,. \tag{5.2}$$

However, in order to find a non-vanishing result the following constraint has to be imposed:

$$N_c = N_f\,. \tag{5.3}$$

Furthermore, we have to impose the D-term constraint for the matter fields. This implies for the VEVs that

$$\Phi_{cf} = \overline{\Phi}^T_{cf} \qquad \text{and} \qquad \widetilde{\Phi}_{fc} = \overline{\widetilde{\Phi}}^T_{fc}\,. \tag{5.4}$$

Let us now consider the bosonic zero mode integration. To lowest order in α', the contributing disc amplitudes with insertion of bosonic zero modes lead to

$$\begin{aligned}
&\tfrac{1}{2}\, b_c \left(\Phi_{cf}\, \overline{\Phi}_{fc'} + \overline{\widetilde{\Phi}}_{cf}\, \widetilde{\Phi}_{fc'}\right) \overline{b}_{c'} + \tfrac{1}{2}\, \overline{\widetilde{b}}_c \left(\Phi_{cf}\, \overline{\Phi}_{fc'} + \overline{\widetilde{\Phi}}_{cf}\, \widetilde{\Phi}_{fc'}\right) \widetilde{b}_{c'} \\
&\qquad = \tfrac{1}{2}\, b_c \left(\Phi_{cf}\, \overline{\Phi}_{fc'} + \overline{\widetilde{\Phi}}_{cf}\, \widetilde{\Phi}_{fc'}\right) \overline{b}_{c'} + \tfrac{1}{2}\left(-\overline{b}_c\right)\left(\Phi_{cf}\, \overline{\Phi}_{fc'} + \overline{\widetilde{\Phi}}_{cf}\, \widetilde{\Phi}_{fc'}\right)\left(-b_c\right) \\
&\qquad = \tfrac{1}{2}\, b_c \left(\Phi_{cf}\, \overline{\Phi}_{fc'} + \overline{\widetilde{\Phi}}_{cf}\, \widetilde{\Phi}_{fc'} + \left(\Phi_{cf}\, \overline{\Phi}_{fc'}\right)^T + \left(\overline{\widetilde{\Phi}}_{cf}\, \widetilde{\Phi}_{fc'}\right)^T\right) \overline{b}_{c'} \\
&\qquad = b\left[\Phi\Phi^T + \widetilde{\Phi}^T\widetilde{\Phi}\right] \overline{b}\,,
\end{aligned} \tag{5.5}$$

where in the last line the constraint (5.4) was employed and we again omitted the VEV brackets. Note that $\Phi\Phi^T$ and $\widetilde{\Phi}^T\widetilde{\Phi}$ are $2N_c \times 2N_c$ matrices. Since there are no bosonic ADHM constraints, the integral over bosonic zero modes to be performed is

$$\int \prod_{c=1}^{N_c} db_c\, d\overline{b}_c\, \exp\left(b\left[\Phi\Phi^T + \widetilde{\Phi}^T\widetilde{\Phi}\right]\overline{b}\right) = \frac{1}{\det\left[\Phi\Phi^T + \widetilde{\Phi}^T\widetilde{\Phi}\right]}\,. \tag{5.6}$$

Combining this result with the one from the fermionic integration (5.2) together with (5.4), one obtains

$$\frac{\det\left[\widetilde{\Phi}^T, \Phi\right]}{\det\left[\Phi\Phi^T + \widetilde{\Phi}^T\widetilde{\Phi}\right]} = \frac{\det\left[\widetilde{\Phi}^T, \Phi\right]}{\det\left[\left[\widetilde{\Phi}^T, \Phi\right]\begin{bmatrix} 0 & \mathbb{1}_{N_c} \\ \mathbb{1}_{N_c} & 0\end{bmatrix}\left[\widetilde{\Phi}^T, \Phi\right]^T\right]} = -\frac{1}{\det\left[\widetilde{\Phi}^T, \Phi\right]}\,. \tag{5.7}$$

Note however, if we define $Q = [\,\widetilde{\Phi}^T, \Phi\,]$ we can bring this result into the following form

$$-\frac{1}{\det Q} = -\frac{1}{\sqrt{\det\left[Q\begin{bmatrix} 0 & \mathbb{1}_{N_c} \\ -\mathbb{1}_{N_c} & 0\end{bmatrix}Q^T\right]}} = -\frac{1}{\mathrm{Pfaff}\,[\,Q\,J\,Q^T\,]}\,, \tag{5.8}$$

where $J = \begin{bmatrix} 0 & \mathbb{1}_{N_c} \\ -\mathbb{1}_{N_c} & 0\end{bmatrix}$. Rescaling the field Q such that it is canonically normalised, collecting all dimensionful parameters and absorbing all numerical factors into Λ the superpotential becomes

$$S_W = \int d^4x\, d^2\theta\, \frac{\Lambda^{2N_f+3}}{\mathrm{Pfaff}\ (QJQ^T)}\,. \tag{5.9}$$

This is precisely the form of the superpotential found by Intriligator and Pouliot in their work [28].

5.2 Orthogonal gauge group

If the orientifold plane has the opposite charge than in the previous case, the gauge group on a stack of N_c D6-branes lying on top of the O6 plane is $SO(N_c)$ and the gauge group on the instanton wrapping the same cycle is $USp(2)$ (In this case, the smallest invariant instanton configuration is given by two E2-branes on top of each other). The flavour brane

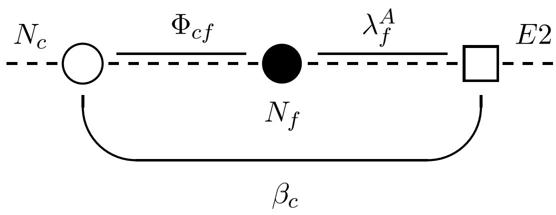

Figure 5: Brane configuration for an instanton in an $SO(N)$ gauge theory. The dashed line represents the orientifold plane.

stack is realised by N_f D6-branes wrapping a three-cycle which is distinct from the one the colour branes wrap, but also invariant under the orientifold projection. The gauge group is assumed to be $SO(N_f)$. There is thus one chiral supermultiplet Φ transforming in the vector representations of both $SO(N_c)$ and $SO(N_f)$.

In the instanton-colour brane sector there are $2N_c$ fermionic zero modes β_c^A and $4N_c$ bosonic zero modes b_c^A, $\bar{b}_c^A$, where $c = 1, \ldots, N_c$ is the $O(N_c)$ index and $A = 1, \ldots, 2$ the $USp(2)$ one. In addition there are $2N_f$ fermionic zero modes λ_f^A $(f = 1, \ldots, N_f)$ from strings stretching between the instanton and the flavour brane. The brane configuration together with all massless modes is shown in the extended quiver diagram in figure 5.

The D- and F-terms are adjoint valued such that in this case there are $3 \times 3 = 9$ bosonic ADHM constraints in agreement with the field theory instanton construction [49]. Analogously, the fermionic ADHM constraints are adjoint-valued, so there are $2 \times 3 = 6$ of them. The fermionic zero mode integral to be evaluated is thus

$$\int \prod_{A=1}^{2} \prod_{c=1}^{N_c} d\beta_c^A \prod_{f=1}^{N_f} d\lambda_f^A \; \delta^{(3)}(b_c^A \sigma^i_{AB} \beta_c^B)\, \delta^{(3)}(\bar{b}_c^A \sigma^i_{AB} \beta_c^B)\, \exp\left(\beta_c^A \Phi_{cf} \lambda_f^B\right). \tag{5.10}$$

Here, the σ^i are the three Pauli matrices. Upon evaluating this integral, one finds the condition $N_f = N_c - 3$ for the generation of a superpotential from instantons. This is in agreement with the field theory result [29].

The explicit evaluation of the various bosonic and fermionic zero mode integrals, including the ADHM constraints, is non-trivial and we leave this for future work. Eventually one should recover the known field theory result for the superpotential [29]

$$S_W = \int d^4x\, d^2\theta \; \frac{\Lambda^{2N_f+3}}{\det[\Phi^T_{fc}\Phi_{cf'}]} \,. \tag{5.11}$$

6. Conclusions

In this paper, for a local intersecting D6-brane configuration giving rise to SQCD with $N_f = N_c - 1$ flavours, we have explicitly computed the one E2-instanton contribution to the superpotential, where the E2-brane lies on top of the stack of N_c colour branes so that in the field theory limit it can be interpreted as a gauge instanton. In the field theory

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limit, we indeed recovered the known ADS superpotential, supporting the point of view that string theory knows about non-perturbative field theory effects. However, when one simply adds such field theory terms to string or supergravity based superpotentials one must keep in mind their limitations. Moving away from the field theory limit, there will appear stringy corrections, which one must control. These are for instance contributions from massive states in the one-loop Pfaffians. Moreover from the string perspective the gauge instanton, i.e. E2 on top of D6$_c$, is only one of the many possible E2-instanton contributions to the superpotential. There can be many other rigid special Lagrangian three-cycles in a Calabi-Yau manifold, which can support E2-instantons. These will also give rise to moduli dependent superpotentials, which cannot be seen in pure field theory. In an honest computation one must either take all these effects into account or must control them in certain limits of the moduli space.

We have carried out the actual instanton computation in quite some detail, firstly to show that such string space-time instanton computations can indeed be performed explicitly and secondly to provide the mathematical means to actually do it. We think that our methods are applicable to more general instanton configurations, either admitting a field theory limit or being completely stringy. It would be interesting to determine ADS like superpotentials for supersymmetric gauge theories with more general matter fields, containing for instance also symmetric and anti-symmetric matter. It would also be interesting to derive explicitly the superpotential induced by gaugino condensation from string theory, i.e. for $N_f < N_c - 1$ in the case of SQCD with gauge group $SU(N_c)$.

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A. Some relations in linear algebra

A.1 General formulae

For the reader's benefit, in this subsection we summarise some well-known formulae which we have used in various places in the main text.

- The derivative of the determinant of a matrix A with respect to the matrix element $A_{i,j}$ is (Jacobi)

$$\frac{\partial}{\partial A_{i,j}} \det A = \det A \cdot \left(A^{-1}\right)_{j,i} \ . \tag{A.1}$$

- The determinant of a block-matrix with mutually commuting matrices A, B, C, D can be computed using the following formula (Strassen)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}B\left(D - CA^{-1}B\right)^{-1}CA^{-1} & -A^{-1}B\left(D - CA^{-1}B\right)^{-1} \\ -\left(D - CA^{-1}B\right)^{-1}CA^{-1} & \left(D - CA^{-1}B\right)^{-1} \end{bmatrix} . \tag{A.2}$$

- If the matrices A, B, C, D mutually commute the following holds

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det\left(AD - BC\right) . \tag{A.3}$$

- For an $N \times N$ matrix the following relation holds, where, as in the main text, $A|_{k,i}$ denotes the matrix A without row k and column i

$$\det A\ \delta_{i,j} = \sum_{k=1}^{N} (-1)^{i+k} A_{k,j}\ \det A|_{k,i} . \tag{A.4}$$

- For an $N \times (N-1)$ matrix A and an $(N-1) \times N$ matrix B it is clear that the determinant of the $N \times N$ matrix AB vanishes:

$$\det AB = 0 . \tag{A.5}$$

- The (i, j)th element of the inverse matrix A^{-1} is

$$A^{-1}_{i,j} = (-1)^{i+j} \frac{\det A|_{j,i}}{\det A} . \tag{A.6}$$

- For $N \times N$ matrices A and B the following relation holds

$$\sum_k \det A|_{i,k} \cdot \det B|_{k,j} = \det AB|_{i,j} . \tag{A.7}$$

A.2 A determinant formula

Let A be an $N \times (N-1)$ matrix and B be an $(N-1) \times N$ matrix. Then define

$$\widetilde{A} = \left[A_{N,(N-1)}, 0_{N,1}\right] \qquad \text{and} \qquad \widetilde{B} = \begin{bmatrix} B_{(N-1),N} \\ 0_{1,N} \end{bmatrix} \tag{A.8}$$

and note that $AB = \widetilde{A}\widetilde{B}$. Then one finds using (A.7)

$$\sum_{i=1}^{N} \det\Big(AB\big((AB+\alpha\,\mathbb{1})^2+\beta^2\,\mathbb{1}\big)\Big)\Big|_{i,i} \tag{A.9}$$
$$= \sum_{i=1}^{N} \det\Big(AB\big((AB)^2+2\alpha AB+\alpha^2\mathbb{1}+\beta^2\mathbb{1}\big)\Big)\Big|_{i,i}$$
$$= \sum_{i,j=1}^{N} \det AB\big|_{i,j}\ \det\big((AB)^2+2\alpha AB+\alpha^2\,\mathbb{1}+\beta^2\,\mathbb{1}\big)\big|_{j,i}$$
$$= \sum_{i,j=1}^{N} \det \widetilde{A}\widetilde{B}\big|_{i,j}\ \det\big((\widetilde{A}\widetilde{B})^2+2\alpha\widetilde{A}\widetilde{B}+\alpha^2\,\mathbb{1}+\beta^2\,\mathbb{1}\big)\big|_{j,i}$$
$$= \sum_{i,j,k=1}^{N} \det \widetilde{A}\big|_{i,k}\ \det \widetilde{B}\big|_{k,j}\ \det\big((\widetilde{A}\widetilde{B})^2+2\alpha\widetilde{A}\widetilde{B}+\alpha^2\,\mathbb{1}+\beta^2\,\mathbb{1}\big)\big|_{j,i}\,.$$

Now note that $\det \widetilde{A}\big|_{i,k}$ and $\det \widetilde{B}\big|_{k,j}$ are only nonzero for $k=N$. Therefore one can write

$$\sum_{i,j=1}^{N} \det \widetilde{B}\big|_{N,j}\ \det\big((\widetilde{A}\widetilde{B})^2+2\alpha\widetilde{A}\widetilde{B}+\alpha^2\mathbb{1}+\beta^2\mathbb{1}\big)\big|_{j,i}\ \det \widetilde{A}\big|_{i,N}$$
$$= \det\Big(\widetilde{B}\big((\widetilde{A}\widetilde{B})^2+2\alpha\widetilde{A}\widetilde{B}+\alpha^2\mathbb{1}+\beta^2\mathbb{1}\big)\widetilde{A}\Big)\Big|_{N,N}$$
$$= \det\Big((\widetilde{B}\widetilde{A})^3+2\alpha(\widetilde{B}\widetilde{A})^2+\big(\alpha^2+\beta^2\big)\,\widetilde{B}\widetilde{A}\Big)\Big|_{N,N}\,. \tag{A.10}$$

Finally, observe that

$$\widetilde{B}\widetilde{A} = \begin{bmatrix} BA & 0 \\ 0 & 0 \end{bmatrix}$$

which then implies that (A.10) is equal to

$$\det\Big((BA)^3+2\alpha(BA)^2+\big(\alpha^2+\beta^2\big)\,BA\Big)$$
$$= \det\Big(BA\big((BA+\alpha\mathbb{1})^2+\beta^2\mathbb{1}\big)\Big)\,. \tag{A.11}$$

This computation shows that (A.9) equal to (A.11).

A.3 A formula concerning characteristic polynomials

Let A be an $N\times(N-k)$ matrix and B be an $(N-k)\times N$ matrix for $k=1,\ldots,(N-1)$. Then define

$$\widetilde{A} = \big[A_{N,(N-k)}, 0_{N,k}\big] \qquad \text{and} \qquad \widetilde{B} = \begin{bmatrix} B_{(N-k),N} \\ 0_{k,N} \end{bmatrix} \tag{A.12}$$

and note that $AB = \widetilde{A}\widetilde{B}$. Furthermore, denote the characteristic polynomial of a matrix M as $\chi_M(\sigma)$ and recall that $\chi_{MN}(\sigma) = \chi_{NM}(\sigma)$ for square matrices M, N. Then it is easy to see that

$$\begin{aligned}
\det\big[AB + \lambda\,\mathbb{1}\big] &= \det\big[\widetilde{A}\widetilde{B} + \lambda\,\mathbb{1}\big] \\
&= \chi_{\widetilde{A}\widetilde{B}}(-\lambda) \\
&= \chi_{\widetilde{B}\widetilde{A}}(-\lambda) \\
&= \det\big[\widetilde{B}\widetilde{A} + \lambda\,\mathbb{1}\big] \\
&= \det\begin{bmatrix} BA+\lambda & & & \\ & \lambda & & \\ & & \ddots & \\ & & & \lambda \end{bmatrix} \\
&= \lambda^k \det\big[BA + \lambda\,\mathbb{1}\big]\ .
\end{aligned} \tag{A.13}$$

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